MODELING THE VALUE OF PROJECT LABOR AGREEMENTS: THE NO-STRIKE CLAUSE OPTION

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I certify that I have read this honors thesis and that, in my opinion, it is fully adequate in scope and quality as an honors thesis for the degree of Bachelor of Science in Mathematics.

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Abstract

This study seeks to explore the area of contract valuation and the issue of no-strike clauses in construction contracts. The belief is that this area represents an extension of the idea of real options and forward contracts for a commodity that is intangible, like a no-strike clause. In a construction contract, Project Labor Agreements (or PLAs) are the method by which labor clauses of a construction project are controlled.This research is motivated by Dr. Jonathan Peters and his analysis on the Building and Construction Trade Council of NY (the organization of construction unions in New York) who were told by a judge in a labor dispute case that no-strike clauses had no value. CCNY and the College of Staten Island worked on developing methods to understand the value that is created by a no-strike or other labor clauses that are controlled through what is called a Project Labor Agreement (PLA). This study hopes to explore a general model to understand the drivers of value and the more specifically the value created by no-strike clauses. No-strike clauses can be extremely lucrative given the safety net they provide in the area of risk management. This research hopes to see a correlation between valuing real options and these no-strike clauses.

Chapter 1

Introduction

The New York City Mayor's office of Contract Services defines Project Labor Agreements (PLAs) as "An agreement by an owner (here, the City) with construction trades that all bidders must agree to as part of a responsive bid. Subcontractors to be used by prime contractors on a City contract must also agree to the terms of the PLA to be approved."[7] In other words PLAs are collective bargaining agreements, whereas all parties involved have a set of distinct requirements for a project that are negotiated. Each PLA is a contract that has a malleable set of requirements, and is meant to be individualized for every contract. After reviewing eight PLAs from New York State and other parts of the country, it was determined that thirteen variables remained a common thread among the PLAs. Table 1 identifies the variables and an example of how each variable may be individualized for a particular project.

The variable that is synonymous with PLAs is the "no-strike clause," in other words an agreement to not halt work on a project in exchange for negotiated terms. The agreement is made to get the "best work for the money with far greater likelihood of on-time, on-budget performance."[3] The terms listed below are negotiated until an agreement is in place that holds both the employer and employees accountable

Type	Common Variables
No Strike Clause	A staple of PLAs
Jurisdictional Disputes	Decided by the employer
Workweek(Days)	5 designated days
Meal Period/Lunch	Half and hour per shift
40 hr Workweek	8 hour shifts, doubling shifts negotiable
Start Times	Between 6am-9am
Overtime	Time and a half
Holidays	8 holidays recognized (Nonnegotiable: Labor Day)
Shift Work	Prior notice required
Reporting Pay	Every 2 weeks
Injury/ Disability	Paid for a full day, on day of injury
Payday	Weekly
Grievances	7 day calendar notice, if not resolved, 7 day arbitration

Table 1.1: Project Labor Agreement Variables.

for a set of previously stated terms within a previously determined time table.[3] This completely transparent time table is what allows the PLA to provide a fair negotiation. Putting in place a provision that protects a project from costly delays can be key when time is a major factor of success in a project. During a delay caused by a strike or a jurisdictional dispute, equipment stands idle while racking up costs for its use and housing. These costs along with others will be examined to determine a value for the No-strike clause in a PLA. In order to provide a value for this no-strike clause it is beneficial to think of it in a similar fashion to those non tangible assets that are traded in futures and forward contracts. Weather options in the weather derivatives market [4] is experiencing increased growth as a number of businesses are facing loss in the midst of bad weather. One area that is benefiting from weather options are energy producing companies. They utilize weather options to "...hedge their risks"[4] with options based on a number of variables that include "... temperature, humidity, rain or snowfall."[4] Similar to PLAs the variables that play a key role in the value of the option are volatile and extremely unpredictable much like the human interactions that could lead to a strike. This study will use the pricing model practices for pricing weather options like in the paper "On Modelling and Pricing Weather Derivatives."[11] One of these practices involves using historical data to observe the past volatility of an option. This methodology will play a key role in the valuation of the no-strike clause "options" formula.

Chapter 2

Background

2.1 Project Labor Agreements

PLAs with a no-strike clause have been used for multiple decades, the first time that PLAs were upheld in court was in 1994 with the renovation of the Tappan Zee bridge.[5] This PLA, which was later discovered to save ”6 Million Dollars”[5], made a serious leap for the acceptance of PLAs in the construction labor force. The acceptance of PLAs has made progress by way of becoming the New York City School Construction Authority's Labor Law Compliance standard. [18] These PLAs have been present with the SCA since 2005, with agreements lasting from 2005-2009, 2010-2014 and 2015-2016. [18] These PLAs are some of the largest and most longstanding public agreements, with the continued implementation of these PLAs standing as a testament to their savings. The use of PLAs for something as time sensitive as the continuous renovation of schools is comprehensible, with the risk of effecting NYC Department of Education calendars and the wellbeing of NYC students. PLAs are perhaps most beneficial to those with these time constraints, and with the number of variables available for negotiation, the individualization of PLAs and the benefits of a no-strike clause can provide an accordance with client and construction company wishes.

2.2 No Arbitrage Pricing

There are a number of ideas to keep in mind when discussing the pricing of options. One of the first is the utilization of a derivative, this is a contract between two parties which derives its value or price from an underlying asset.[14] The buyer of the contract or "option" acquires the right (but has no obligation) to buy or sell the option at an agreed upon price at a future time.[1] Two alternate forms of this option are "call" and "put" options. The latter invokes the right for the the holder of the option (not the obligation) the to sell a stock for a price, K (the strike price) at a future time. A call option will give the holder of the option the right to buy stock for a particular price. Call and put options are only exercised when a stock is "in the money" in other words, when it begins to have value for the option holder. For a call option this occurs when the price of a stock rises, $(S_T - K)^+$ where S_T is the price of the stock at the exercise date.The put option is exercised when the stock price falls,$(K - S_T)^+$.[1] Another way to represent this is:

$$(S_T - K)^+ = max(S_T - K, 0)$$

$$(K - S_T)^+ = max(K - S_T, 0)$$

With the idea of derivatives explained, let's explore one of the most common contracts, forward contracts. Forward contracts are customized contracts between two parties to buy or sell an asset at a specified price on a future date.[15] These parties involved go into the contract privately and decide upon a price with a predetermined volatility, set for one date in the future to purchase and sell the asset.

Forward contracts only involve the parties that are buying and selling the options, that is, there is no regulation to ensure that neither party will default. [15] Should this contract have involved a third party (a clearing house) to guarantee the delivery of this asset the risk would be lowered but the customization on delivery date, asset and delivery method would be restricted.

The pricing of these options described above invokes the common thread among these models, that is a no-arbitrage pricing method. Arbitrage is the "simultaneous purchase and sale of an asset in order to profit from a difference in the price."[16] This is essentially utilizing the market that these transactions take place in and manipulating the stock and bond interactions to make something from nothing. In an efficient market this should not be possible. As one will see with the next few sections, widely accepted pricing models that are to be correct must be fitted with an arbitrage free component to ensure a fair and "risk-free" price.[1]

2.3 Black-Scholes Model

The Black-Scholes model for pricing derivatives has been a key player in determining the value of options in both the American and European financial markets. The Nobel Prize winning formula has been used for over a century, it was made public in 1973 its creators, Robert Merton, Myron Scholes, and Fischer Black.[9] The formula has been widely used since then and gave great success to hedge funds like "Long Term Capital Management."[9] The Black-Scholes formula relies on a number of conditions to be successful when the determining the value of an option. Foremost, the arguably most important variable in the formula is the options volatility; as variables in the Black-Scholes formula go, this is the most individualized part of the formula. It is largely historically based and is determined by the up and down trends of the price of a stock, with a trend observed and noted the stock's "average" value is determined. Volatility is counted on the fact that deviations from the mean will be most likely closest to the mean. Lets observe a Facebook stock trading for 30 dollars for example, that is typically traded with a volatility of 0.03, in turn a change of 90 cents is expected with this option.

An indispensable part of this formula is that major deviations from the mean are not what is to be expected, instead, historical changes based on normal market conditions are to be expected. One way this is evident is with the physics approach to the Black-Scholes formula[9], that is, its key component is the normal distribution that is used. The bell curve is widely used in academia as the natural tendency of many predictable events to occur, most notably "...the most likely paths that something buffeted by these randomly moving particles [atoms]."[9] By considering the movement of atoms similar to that of changing stock prices it supports the widely accepted methodology involved in the Black-Scholes formula, that stock prices, and in effect the value of the options that stem from them, are random events. It should be noted that the Black-Scholes formula is widely accepted not due to the seamless nature of the calculation, instead, it is used under the prospect that, "Everything that is tweaked, however, leads to more issues. Today, there is no clear successor to the BS model."[12] The inputs of Black-Scholes do posses a number of limitations that should be noted. The first being that the model itself is designed for valuing an option that can only be exercised at the date of expiration, in other words a "European option" with no dividend opportunity.[13] An American option can be exercised at any point before the exercise date and dividends are inevitable.[13] One of the last assumptions that are made in the Black-Scholes model is that of a constant volatility for the life of the option. This is a generalization made that allows for a point estimate of a sample space of strike prices for a historical period of the stock. The variance of the historical data collected is inputted as in the equation. Volatility plays a key role in the development of the formula, including the binomial tree equation for the portfolio that Black-Scholes aims at replicating.

The Black-Scholes formula utilizes a number of variables from the underlying asset in question, to determine the fair price of the call or put option. As explained in the formula below, five measures of the option are required. It is important to note that the time variable in the formula, T is in days. The number of days until expiration is to be converted as necessary, with 365 as the denominator (or 252, accounting for the actual number of trading days in a year).
Black-Scholes Formula (European Call Option)

$$\begin{aligned} V &= E_Q\Big[\Big(S_0 e^{\sigma\sqrt{T}Z-\frac{\sigma^2}{2}T} - ke^{-rT}\Big)^+\Big] \\ &= S_0\Phi\Big(\frac{ln(S_0/k)+(r+\frac{\sigma^2}{2})T}{\sigma^2\sqrt{T}}\Big) - ke^{-rT}\Phi\Big(\frac{ln(S_0/k)+(r-\frac{\sigma^2}{2})T}{\sigma^2\sqrt{T}}\Big) \\ &= S_0\Phi(d_1) - ke^{-rT}\Phi(d_2) \end{aligned}$$

2.4 Derivation of the Black-Scholes Formula

As stated before there are a number of assumptions made to derive the Black-Scholes formula. Firstly, this is a European call option, as such the date of expiration is the only time the option may be exercised. This assumption is important later on when discussing the δt as it pertains to the derivation of the Black-Scholes formula. The next assumption follows the goal for options pricing models to price options with no arbitrage and in turn a risk neutral situation. The final assumption of the Black-Scholes formula, for this derivation, is that the risk free rate is equal to zero. For this derivation we will consider a call option.

We must first consider that in the first interval of time, t=1 a stock has the option to go in two directions, it can go up in price (u) or down (d).In this discrete situation stock processes follow a binomial distribution. Following a number of up and down movements it can be considered that n is the number of up and down movements the stock follows and the number of successes p=instances of u (since this option is a call, these derivative are "in the money" and exercised when the price of the stock goes up) and the number of failures q=instances of d. Both of the directions the stock may take posses the same probability of 1/2 [1].

Now, as we discussed, this binomial property of stock movement considers a discrete number of n, in terms of binomial distribution, between two data points within a time t there is no value between the two. In the world of stock movement within a time interval there are a (in theory) infinite number of movements a stock can take. This leads to the consideration of stock movement in the continuous sense, by which every two data points would have a data point between them. With the number of n "increasing" to infinity the distribution of the stock data is continuous and therefore normal by the Central Limit Theorem [1].

With normal distribution in mind it is now time to observe the idea of random walks. We consider a random walk, W_n in time interval [0,1] with n steps. This time interval allows for only a discrete form of movement for the stock, with $\delta t = 1/n$. However, now that the stock process is defined as normally distributed the random variable generator integral to the Black-Scholes Model, Brownian motion, W_t, can come into play. This new continuous from of random walks utilizes the convergence of W_n to Z where $Z \sim N(0,1)$ Brownian motion is the "... idealization of the trajectory of a single particle being constantly bombarded by an infinite number of infinitesimally small random forces."[21]. Brownian motion considers the random walks or "up and down" movements by the measure to be infinite with a continuous time interval. W_t will generate a random variable that is normally distributed,

$$W_t \sim N(0,t)$$

where mean is equal to 0 and variance is t.

With a continuous random variable generator now found, the next step in deriving the Black-Scholes Formula calls upon the notion that this call option is to be valued under a risk neutral measure with no arbitrage, that is, no opportunity for "free money" where the risk free rate is equal to zero. The goal with the Black-Scholes Formula is to price an option in a market that maintains equilibrium.[1] In order to achieve this arbitrage free pricing model we must observe the transition between a stochastic differential equation (SDE) with exponential Brownian motion to one that also possesses a risk neutral measure to eventually produce the arbitrage free Black-Scholes model to evaluate an option.

Let us first borrow the SDE with exponential Brownian motion (for this derivation we assume that the interest rate $r = 0$):

$$S_t = S_o e^{\mu t + \sigma W_t}$$

Using Ito's Lemma, we find

$$dS_t = S_t \left(\mu dt + dW_t + \frac{\sigma^2}{2} dt \right) = \sigma S_t \left(\frac{\mu + \sigma^2/2}{\sigma} dt + dW_t \right)$$

Let

$$d\tilde{W}_t = \frac{\mu + \sigma^2/2}{\sigma} dt + dW_t$$

This step now calls for a change of measure, one that not only provides a risk neutral measure but one that removes the drift μ from our formula. To do this one must invoke the Cameron-Martin-Girsanov theorem which allows us to construct a martingale measure. A martingale is a characteristic given to a stochastic process where under the filtration of a sub process F_s a process X_t has an expected value of X_s based off of that stock portfolio (up and down jumps) determined by that filter [1]. This martingale may only exist if the drift in the SDE is equal to zero and follows that

$$E(X_t|F_s) = X_s$$

With this criteria for martingales we must now find the γ that will create this martingale and in effect take away the drift in the measure μ.
Recalling that

$$d\tilde{W}_t = \frac{\mu + \sigma^2/2}{\sigma}dt + dW_t$$

We invoke Girsanov and have

$$\gamma = \frac{\mu + \sigma^2/2}{\sigma}$$

and borrow that [16]

$$\tilde{W}_t = \gamma t + W_t$$

$$\tilde{W}_t - \gamma t = W_t$$

plugging back into S_t

$$S_t = S_o e^{\mu t + \sigma(\tilde{W}_t - \gamma t)}$$

$$S_t = S_o e^{\mu t + \sigma\tilde{W}_t - (\mu + \frac{\sigma^2}{2})t}$$

Combining positive and negative drift terms will take away the drift, we now have a stock process S_t (with a risk neutral rate of 0) where $\mu = 0$, hence, producing and arbitrage free model and

$$S_t = S_o e^{\sigma\tilde{W}_t - (\frac{\sigma^2}{2})t}$$

Note that $\tilde{W}_t$ is a Q-Martingale Brownian motion. Finally, we borrow that

$$dS_t = \sigma S_t \tilde{W}_t$$

the value of a call option is

$$V = E_Q((S_t - K)^+)$$

the Black-Scholes formula.

2.5 Volatility

Volatility is a complex measure that, as in the case with the Black-Scholes formula, is used to determine the fluctuations of the market and how they will effect the asset of the option at the time of expiration.[1] In variations of the Black-Scholes formula, volatility of the option is denoted as sigma, σ, a symbol that is commonly used in statistics as the standard deviation of a set of data. The standard deviation, when used as a statistical measurement, can also be a contributor to historical volatility.[1] Standard deviation, most simply put, is the dispersion from the mean. With historical volatility, the variations from the expected average price of an option based on historically collected and analyzed data can provide insight into the level of risk of an option. This measure of risk is essential and often times ambiguous calculation, the historical volatility is used as a stand-in for an expected volatility since predictions of the tendencies of an option are most widely associated with its history, the same could be said for strikes by union construction workers in the labor force. The juxtaposition of options of an underlying asset in the forms of calls and puts, to the risk deterrents of a no-strike clause leads to impending questions about volatility and its use and assumptions in models like the Black Scholes.

With a historical approach being the forthright path it is probable to assume that volatility is calculated from the relative prices of an asset. More specifically, "the difference in log prices, which will be normally distributed" [1] taking the natural log of an earlier price and dividing by the current price. Properties of logs establish that this division is in fact a difference between the logs of each price.

$$\ln\left(\frac{\text{Price}_1}{\text{Price}_2}\right)$$

With the Black-Scholes model, the d_1 or cumulative probability calculation utilizes this with $Price_1 = S_0$, Stock Price at time zero (current stock price) and $Price_2 = k$, Strike Price.

$$\ln\left(\frac{S_0}{k}\right)$$

Distributing this difference over normal distribution determines the number of standard deviations from the mean, or in this case the average stock price. This normalization that takes place references the Black-Scholes formula once again where the cumulative probability is found for two standard normal points,namely, d_1 and d_2. The process of finding the volatility and distribution for our no-strike clause model and more will be discussed further in this paper. First, we must observe a potentially analogous model and delve into the weather derivatives market.

Chapter 3

Weather Options

3.1 Non-Tangible Asset

The weather derivatives market is a growing entity with a "more liquid and dynamic hedging of weather options" [4]. With the lack of a predictable volatility in the futures sense, whilst still remaining within a historically based expected realm. Like the predictions made in the Black Scholes formula, weather options do not usually take into account the "once in a life time" weather catastrophes. However, anything that could effect the delivery of other products such as crops or energy do take into account seasonally present events like hurricanes. The Chicago Mercantile Exchange is a particular market that takes pride in "offering multiple risk management opportunities related to temperature, snowfall, frost, rainfall and hurricanes."[19] This risk management can provide a number of safeguards for these energy companies, with peace of mind that a delayed delivery will not mean zero profit. Now let us look at the cost of this peace of mind in a similar manor to that of the Black-Scholes formula. That is, we will borrow the methodology and models from Financial Analyst Peter Alaton, and Mathematician's Bounalem Djechiche and David Stillberger's paper "On

Modelling and Pricing Weather Derivatives." This paper finds a pricing model for weather derivatives on the basis of temperature [11] and we will discuss the methods and results of this paper in the following sections.

3.2 Distribution, Martingales, and Trajectories: The Temperature Model

In regard to weather options one of the most traded upon attributes is temperature. Temperature, unlike an out of season hurricane, is fairly predictable based on historical data. Another benefit of observing temperature is that most likely the variance of temperature is not large, in the respect that the deviations from the mean are not large within a small interval of time. Taking into account that temperature usually reverts toward the mean, a good candidate to model temperature fluctuations is an Ornstein-Uhlenbeck process [17] which is written as

$$dT_t = a(T_t^m - T_t)dt + \sigma_t dW_t$$

Note that T_t^m is the mean temperature at the time t and dW_t represents the Brownian increment. In order to revert the mean for this SDE in regards to temperature, the term $\frac{dT_t^m}{dt}$ must be added to the drift term. According to Alaton [11] a possible choice is

$$\frac{dT_t^m}{dt} = B + \omega C \cos(\omega t + \varphi)$$

The above will act as an adjuster to the non-constant mean temperature and the full model is then written as

$$dT_t = \left\{ \frac{dT_t^m}{dt} + a(T_t^m - T_t) \right\} dt + \sigma_t dW_t$$

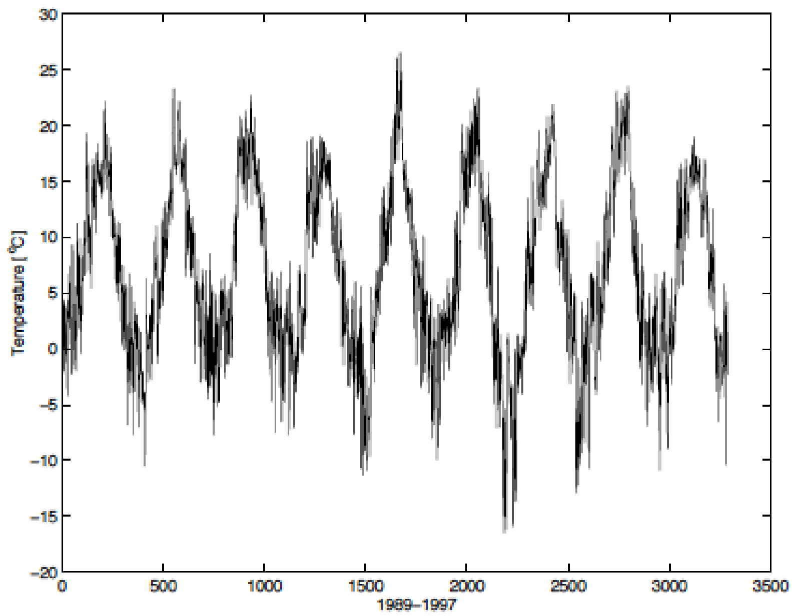

Figure 3.1: Graph of temperature fluctuations at Bromma aiport (source: [11]).

As a particular example discussed in Alaton's work [11], the authors look in detail at at Bromma Airport in Stockholm, Sweden, during 1989-1997. The following figure 3.1 shows the temperature fluctuations over a span over 40 years.

From there, it was determined that $T_t^m = A + Bt + C\sin(\omega t + \varphi)$ can be fitted to the data (by use of a parameter vector in the method of least squares). That is, squaring the difference of the vector with elements from the equation and a vector of elements from the data obtained $\xi = (a_1, a_2, a_3, a_4)$. The constants of the model (A,B,C,φ) are derived from this data to get a function for the mean temperature, T_t^m. With the given parameters this formula produces

$$T_t^m = 5.97 + 6.57 \times 10^{-5} t + 10.4 \sin\left(\frac{2\pi}{365} t - 2.01\right)$$

The standard deviation, σ, estimation is developed from the data set on a monthly analysis with each month, μ,with days n taken separately and "discretised" into an equation of regression, where a "regression of today's is on yesterday's temperature" [11]. This estimator is

$$\hat{\sigma}_\mu^2 = \frac{1}{N_\mu - 2} \sum_{j=1}^{N_\mu} \left(\tilde{T}_j - \hat{a} T_{j-1}^m - (1 - \hat{a}) T_{j-1} \right)^2$$

Now that the mean temperature formula, constants, and formula for the estimator of σ have been derived, the final component is the estimator of the mean-reversion parameter, a, which is used throughout. A martingale estimation factor to estimate this parameter is utilized with expected value of temperature T_i being filtered on the temperature that came before it T_{i-1}:

$$E[T_i | T_{i-1}] = (T_{i-1} - T_{i-1}^m) e^{-a} + T_i^m$$

This martingale is entered into the formula for the estimator of a which is denoted $\hat{a_n}$ [13]. In order to find this one must do so in the from of the "martingale estimation functions method.

$$G_n(\hat{a_n}) = 0$$

Where n is the number of days in the month and

$$G_n(a) = \sum_{i=1}^{n} \frac{T_{i-1}^m - T_{i-1}}{\sigma_{i-1}^2} \{ T_i - (T_{i-1} - T_{i-1}^m) e^{-a} - T_i^m \}$$

Further analysis and implementation of the data set produce an $\hat{a_n}$ which allows the temperature model to "...have the same properties as the observed temperature." [11] This model is able to simulate trajectories of the observed temperature spanning the

40 year data set.

3.3 Market Risk, Expected Value, and Variance

As explained in the first section of this chapter, weather derivatives are not able to be priced by the traditional method as with tangible assets. Instead, weather derivatives are priced on risk. The "market price of risk, λ" which is to be assumed constant as a result of no true market for risk developed.[11] As with the Black-Scholes model derivation assets for the weather markets are to be assumed to be risk free and possessing a constant interest rate of r (note that in our simplified derivation we gave r the value of zero). Q-Martingales remain a major part of this pricing model, where V_t is the new risk neutral Q-measure. The "price process" T_t maintains the following

$$dT_t = \left\{ \frac{dT_t^m}{dt} + a(T_t^m - T_t) - \lambda\sigma_t \right\} dt + \sigma_t dV_t$$

Where $t \geq 0$. The change of measure above is conducted using a Girsanov transformation, in order to provide that risk neutral measure, in an analogous fashion to the Black-Scholes Model. The price of an option is based on the expected value and variance, and both have been derived and generalized for large time intervals.[11] This takes into account the use of integrals that are split into the sum of two integrals to find the expected value and variance between two time ticks. In the case of weather options this can be done for the difference of temperature between the first day of the month and the last. This interval can be broken up into smaller intervals with its overall count increasing (similar to the n "increasing" to infinity with the Black-Scholes model). The price process expected value is represented by $E^Q[T_t|F_s]$ and variance $Var[T_t|F_s]$.

3.4 Derivative Pricing: Weather Market

In the weather derivatives market there is a standard of temperature derivatives based on "heating or cooling degree days" [4] represented as HDD and CDD respectively. The paper that the pricing model is based off of uses an HDD call option to exemplify how to price weather derivatives. We should first observe that HDD is the number of degrees (in degrees Celsius) that a temperature will vary from for the day. Heating degree days usually take place from November to March as that is when temperatures are coldest and the highest likelihood of energy usage by families (recall that energy distributors are the some of the largest to benefit from weather derivatives). [11] The temperature standard in the U.S. (and utilized in European markets) is 18° Celsius (65° Fahrenheit). HDD is generated by

$$HDD_i = max\{18 - T_i, 0\}$$

and alternatively, CDD

$$CDD_i = max\{T_i - 18, 0\}$$

Note that the generators of these values resemble that of non-weather derivative markets, where the date of expiration decision to be exercised is based on the maximum difference of strike price and stock price (see 2.2).

We borrow that HDD payout is done by these two methods

$$\chi = \alpha \max\{H_n - K, 0\}$$

$$H_n = \sum_{i=1}^{n} \max\{18 - T_t, 0\}$$

where α=1 unit of currency/HDD.

Under the Q-martingale risk neutral measure it is noted that $T_t \sim N(\mu_t, V_t)$, on a particular winter day where H_n is an accumulation of the month's temperatures we know that T_t is a Gaussian or normal process by the previous sections. Borrowing from Alaton [11],

$$E^Q[H_n|F_t] = \mu_n$$

$$Var[H_n|F_t] = \sigma_n^2$$

as such, the distribution of H_n is $N \sim (\mu_n, \sigma_n)$. The final result as the call HDD option pricing formula is

$$c(t) = e^{-r(t_n-t)} E^q[max\{H_n - K, 0\}|F_t]$$

$$= e^{-r(t_n-t)} \int_0^{\infty} (x - K) f_{H_n}(x) dx$$

$$= e^{-r(t_n-t)} \Big((\mu_n - K)\Phi(-\alpha_n) + \frac{\sigma_n}{\sqrt{2\pi}} e^{-\frac{\alpha_n^2}{2}} \Big)$$

where $\alpha_n = (k - \mu_n/\sigma_n)$. Like the Black-Scholes formula, thanks to the normal distribution of the pricing formula, the cumulative distribution is also found here. Unlike the Black-Scholes formula, the temperature pricing model still allows for the drift term, μ_n to play a role in the formula hence not producing a no-arbitrage model. This leaves room for speculative betting and unlike Black-Scholes does not prohibit the availability of free money. However, unlike the trading of tangible assets the weather derivatives market is based solely on risk and betting, so this distinction is rather probable.

Chapter 4

The No-Strike Clause Option

Now that we have explored two models that play key roles in derivative pricing, the first being the industry standard of Black-Scholes and the second being the temperature derivatives pricing model, it is in this chapter that we will provide examples for the pricing of the "No-strike clause" of Project Labor Agreements.

We will approach this pricing with the idea that each day of a strike will cost the owners of a project the equivalent value of those days not on strike. The price will be a percentage of the yearly money spent (in this case salary). Borrowing from the Bureau of Labor Statistics we will use the term "days idle" to describe these days where work has been stopped; the BLS utilizes data for "work stoppages" with the following characteristics: they are large work stoppages with $> 1,000$ workers involved, they include both strikes and employer initiated lockouts, they must last at least 1 shift.[22]

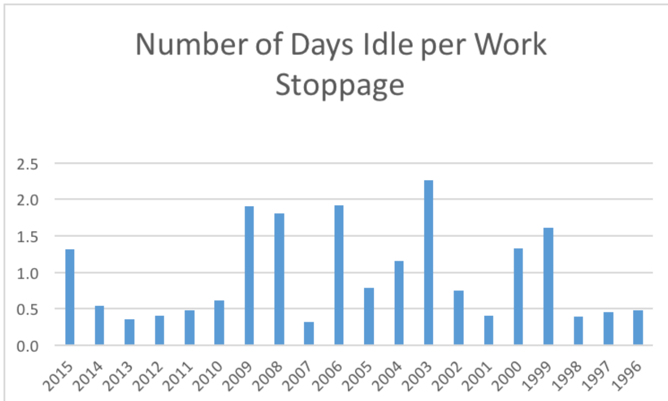

Figure 4.1: Number of days idle per work stoppage. (source: [22]).

4.1 Work stoppages data

Utilizing the work stoppages data from the BLS, here is a table examining the average number of days idle per work stoppage occurrence by year. We will use the last 20 years as a reference. (i.e. 1996-2015)

For this study we will calculate a number for the number of days idle in a given year for a strike. To do this we take the average of the number of strikes per year, 20.25, and then find the average number of days idle per each strike, 0.96, multiplying the two, we get that the average number of days spent idle in a given year is 19.51. We must now normalize this.

We will be using the percentage of the number of working days in the New York City calender which is 252 days. This will be used as the percentage of yearly money spent which will act as a normalization for the potential salary lost on the days idle from striking. Which will in turn be the expected loss due to striking. Taking the number of days idle over the number of working days in a calendar year,

$$\frac{19.51}{252} = 7.74\%$$

the percent of the number of days idle for a strike in a given year is 7.74%. We may also look at the number of days idle per one strike in a given year,

$$\frac{0.96}{252} = 0.38\%$$

with less than 1/2 a percent per day.

In order to determine the cost savings from a no-strike clause we will use the data above to develop a method for the percentage for a potential loss of profit from a strike. This will allow for the determination for cost savings and/or how expensive including a no-strike clause may be for a project. To do this, we must determine the expected loss for a project and this will be done assuming two distributions in the discrete form, Uniform distribution and Poisson distribution.

4.2 Probability Distribution

To calculate the expected loss we will first consider the loss using Uniform Distribution. We will utilize the property of the expected value, or mean, of this distribution

$$\sum_{i=1}^{n} x_i p_i$$

where x_i is the loss from strike in one year (by working days= 252). And the p_i will be equal to the uniform distribution with the denominator of each probability or p_i given by the maximum, b, and minimum, a, to strike days as determined by the data examined in Figure Three. For simplicity we will mirror the methods of Black-Scholes and make a number of assumptions.

First, we will be observing the discrete case of one year with 252 working days, the expected loss calculated from the occurrence of one strike with a varying number of

Days per year	p_i
1	1/3
2	1/3
3	1/3

Table 4.1: Uniform Distribution: Days Idle.

days of duration. Table 4.1 depicts the range of one strike duration from data taken over the past twenty years. Each data point was rounded up to whole days.

Now that we can see clearly the probability of each strike duration, we must find the loss or, x_i, of each strike duration.

$$x_1 = 1 \cdot (0.0038) = 0.0038$$

$$x_2 = 2 \cdot (0.0038) = 0.0076$$

$$x_3 = 3 \cdot (0.0038) = 0.0114$$

Inputting the probabilities into our formula for $E(x)$.

$$E(x) = 0.0038 \cdot (1/3) + 0.0076 \cdot (1/3) + 0.0114 \cdot (1/3)$$

$$E(x) = 0.0013 + 0.0025 + 0.0038$$

$$E(x) = 0.0076$$

The expected loss per strike in one year for a project is 0.76% of earnings or in this case salary, when assuming Uniform Distribution.

The second distribution we will consider for the determination of loss by the varying duration of a strike is Poisson Distribution. The "days per year" variable will be the same, and the calculation of the expected value will follow

$$E(x) = \sum_{n=1}^{\infty} np(n)$$

$$E(x) = \sum_{n=1}^{\infty} n \cdot \frac{e^{-\lambda}\lambda^n}{n!}$$

Where by properties of the Poisson Distribution,

$$E(x) = \lambda$$

For this Poisson Distribution determination of $E(x)$ we will be using the probability of a strike for one year, 0.0038. Therefore the expected value for a strike in a given year is,the sum of the probability for a strike,0.0038, multiplied by the days idle.

$$E(x) = 1 \cdot (0.0038) + 2 \cdot (0.0038) + 3 \cdot (0.0038)$$

$$E(x) = 0.0038 + 0.0076 + 0.0114$$

$$E(x) = 0.0228$$

In deciding which distribution would be the most ideal for our model, we will take a look at a relative frequency histogram for the last twenty years of work stoppages.

This histogram does not display the visual representation of either distribution (Uniform or Poisson) which results in the ideal distribution not being found. We have explored two possible distributions to be used for the pricing of an option in the world of Project Labor Agreements. The no-strike clause model we have considered will take into account the loss of each day of a strike as a proportion of the salary and/or project costs. This customization ability goes hand in hand with PLAs as one

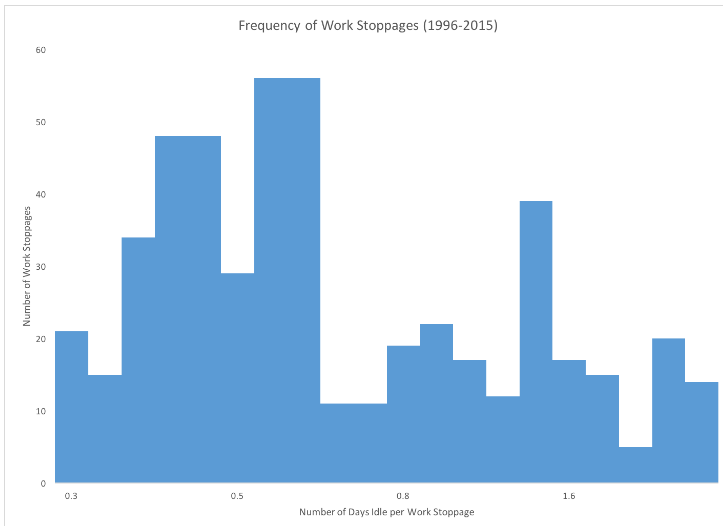

Figure 4.2: Relative frequency histogram for the last twenty years of work stoppage. (source: [22]).

of the key factors of appeal with construction contracts.

Chapter 5

Conclusion: Value of the no-strike clause

This study sought to explore the areas of contract valuation and determine if a value for a no-strike clause can be found. With two distributions considered and the models for a comparable non-tangible asset examined, the value that comes from a no-strike clause possess two properties. The first property, much like weather options, owners of a PLA are ultimately purchasing an insurance policy, the cost of that peace of mind can vary depending on the needs of each project owner. In order to develop a method by which a value can be found for such volatile methods, our model has maintained a method of comparable analysis. In our model the value of a no-strike clause equal to the expected value of that strike multiplied by the salary of a projects employees. By uniform distribution the expected value is $0.0076(S)$ where $S = salary$ and by Poisson, $0.0228(S)$. Therefore, the owner of a contract has the ability to save either 0.76% in salary costs for each day idle in the case of a strike, or 2.28%. Depending on the scope of the project, the number of trades involved and the significance of time on a project, these differences can mean quite a lot in the field. Project Labor

Agreements can provide a range of safeguards for both the owners of a project and the contractors involved, the value of these contracts vary greatly. Following this analysis of the no-strike clause, there is a potential value of this clause in the project involved, whichever distribution an expected value is calculated for. The value of a no-strike clause, and in turn a peace of mind on striking, is equated to a number of missed days. These missed days and altered schedules are all within prevention, that is, at the right price.

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